SATURATION ION CURRENT TO AN ELECTRIC PROBE IN A SLOWLY MOVING PLASMA

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The subject of this paper is the saturation current to an electric probe washed by a stream of moderately ionized plasma with low Reynolds number of the incident flow. It is known that the problem of determining the saturation current reduces to calculating the quasineutral molar concentration of charged particles [1].

A formula has been obtained for the saturation current to a spherical probe, allowing for the plasma being nonisothermal and having variable transport properties. To solve this problem in the limit of low Reynolds number of the incident plasma stream we use the method of matched asymptotic expansions. This method was used in [2, 3] to find an expression for the flux of particles diffusing to a body of arbitrary shape in an isothermal medium. In the present paper the approach used in [2, 3] is modified to obtain an expression for the flux particles diffusing to a sphere in a nonisothermal medium with variable transport properties.

From this formula and the previously known theoretical and experimental results, the conclusion is drawn that in a certain range of variation of probe temperature the saturation ion current to an electric probe in a chemically frozen plasma depends only slightly on this temperature. By neglecting this dependence and using the results of [2, 3] we obtained a formula for the saturation current to a probe of arbitrary geometrical shape in a plasma flux at low Reynolds number.

The "moving probe" method suggested in the experimental work of [4] for diagnosis of a slowly moving plasma has been confirmed theoretically and developed.

1. Statement of the Problem. We consider flow of a three-component moderately ionized gas (plasma, consisting of singly charged ions of one type, electrons, and one type of neutrals) about a conducting charged body (an electric probe), under conditions where the particle mean free path is much less than the characteristic probe dimension. We shall assume the plasma flow to be in thermodynamic equilibrium (the temperatures of the electrons and of the heavy particles are the same) and chemically frozen, with heterogeneous reactions on the probe surface.

We define the dimensionless saturation ion current I_i to an electric probe of arbitrary geometric shape in the form

$$I_i = -I_i^0/(en_{e\infty}^0 D_{in\infty}^0 L),\ I_i^0 = \int_S \mathbf{j}_i^0 d\mathbf{S}^0, \tag{1.1}$$

where $\mathbf{j}_i^0$ is the saturation ion current density to the probe; the surface element $d\mathbf{S}^0$ is directed along the outward normal to the probe surface S; e, charge on the electron; n_e^0, volume concentration of electrons; D_{ij}^0, binary diffusion coefficient; L, characteristic probe dimension (for a spheric probe we take this to be the probe radius a); the subscripts i, e, and n refer to ions, electrons, and neutrals, respectively; the subscript ∞ corresponds to incident stream conditions.

Below, the dimensioned physical quantities, in contrast with the corresponding dimensionless quantities, will be denoted by the subscript 0.

Substituting into Eq. (1.1) the expression for the saturation ion current density found in [1], we obtain, in dimensionless variables

$$I_i = 4\pi(\rho D_{in})_w \mathrm{Sh},\ \mathrm{Sh} = -\frac{1}{2\pi}\int_S \nabla\xi d\mathbf{S}, \tag{1.2}$$

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where ρ, D_{in} are the dimensionless density and diffusion coefficient; $\xi = (x_\infty - x)/x_\infty$; x, quasineutral molar concentration of charged particles; Sh, dimensionless Sherwood number, describing the flow of charged particles to the probe; the subscript w refers to conditions at the probe surface. Here and below we take the length scale to be the characteristic probe dimension L, while the other quantities are scaled in terms of their values at infinity (densities, temperatures, transport coefficients, velocities). The quantity ρD_{in} is assumed to be constant at the probe surface because its surface has constant temperature.

The function ξ is determined the ambipolar diffusion equation [1], which, assuming negligible effect of thermal diffusion and neglecting the barodiffusion term that is quadratic in the Mach number, has the form, in dimensionless variables

$$\mathrm{Re}_\infty \mathrm{Sc}_\infty \rho \mathbf{v} \nabla \xi - \operatorname{div}(\rho D_a \nabla \xi) = 0,\ \mathrm{Re}_\infty = \left(\frac{\rho^0 v^0 L}{\mu^0}\right)_\infty,$$

$$\mathrm{Sc}_\infty = \left(\frac{\mu^0}{\rho^0 D_a^0}\right)_\infty,\ D_a^0 = \frac{2D_{in}^0 D_{en}^0}{D_{en}^0 + D_{in}^0} \approx 2D_{in}^0,$$

where $\mathbf{v}$ is the bulk velocity, referenced to its value at inifinity v_x^0; Re and Sc, Reynolds number and the ambipolar Schmidt number; μ^0, coefficient of dynamic viscosity.

Assuming the Schmidt number to be constant and a power law for the viscosity, the last equation has the form

$$\mathrm{Re}_\infty \rho \mathbf{v} \nabla \xi - (1/\mathrm{Sc}) \operatorname{div}(\mu \nabla \xi) = 0,\ \mu = T^n, \tag{1.3}$$

where T is the temperature.

We assume the probe surface to be perfectly absorbing, perfectly catalytic, and nonemittive. In this case one can take the quasineutral concentration of charged particles on the wetted surface to be zero [1]. Far from the probe the ion and electron concentrations tend to their values in the unperturbed plasma. Therefore, the boundary conditions for the function ξ have the form

$$\text{on the probe surface } \xi = 1, \text{ far from the probe } \xi = 0. \tag{1.4}$$

The quantities $\mathbf{v}$, ρ, T are determined by solving the problem of flow over a body (the probe) of a viscous thermally conducting gas, without allowing for ionization.

As was mentioned above, the objective of this paper is to determine the form of the function $I_i(\mathrm{Re}_\infty)$ for small values of the argument Re_∞. More accurately, we shall seek the first two terms of the expansion of this function as $\mathrm{Re}_\infty \to 0$.

2. Spherical Probe in a Nonisothermal Plasma. To solve the problem of a spherical probe in a nonisothermal slowly moving plasma, besides Eqs. (1.3) and (1.4) we require the heat influx equation and the continuity equation, and also the boundary conditions for the temperature and the bulk velocity.

The heat influx equation [5], neglecting terms on the order of the square of the Mach number, and the boundary conditions for the temperature have the form

$$\mathrm{Re}_\infty \rho \mathbf{v} \nabla T - \frac{1}{\sigma} \operatorname{div}(\mu \nabla T) = 0,\ \sigma = \frac{\mu^0 c_p^0}{\lambda^0}, \tag{2.1}$$
$$r = 1,\ T = T_w;\ r \to \infty,\ T \to 1,$$

where σ is the Prandtl number; λ^0, thermal conductivity; c_p^0, specific heat; and r, radial coordinate. Here it is assumed that the neutral gas has constant heat capacity and Prandtl number.

Following [2, 3], we shall seek an approximate asymptotic solution of the problem of Eqs. (1.3), (1.4), and (2.1), using the method of matched asymptotic expansions with respect to Reynolds number in the inner $[1 \le r < O(\mathrm{Re}_\infty^{-1})]$ and outer $[O(\mathrm{Re}_\infty^{-1}) \le r < \infty]$ flow regions.

The asymptotic expansions of the solution of the problem have the form:

inner

$$\xi(r, \theta; \mathrm{Re}_\infty) = \xi_0(r, \theta) + \mathrm{Re}_\infty \xi_1(r, \theta) + \ldots, \tag{2.2}$$
$$T(r, \theta; \mathrm{Re}_\infty) = T_0(r, \theta) + \mathrm{Re}_\infty T_1(r, \theta) + \ldots;$$

outer

$$\xi(r, \theta; \mathrm{Re}_\infty) = \mathrm{Re}_\infty \xi^{(1)}(r', \theta) + \ldots, \tag{2.3}$$
$$T(r, \theta; \mathrm{Re}_\infty) = 1 + \mathrm{Re}_\infty T^{(1)}(r', \theta) + \ldots,$$

where θ is the angle between the radius vector r and the direction of the incident stream velocity; $r' = r\,\mathrm{Re}_\infty$ is the compressed radial coordinate.

We shall postulate that the outer and inner expansions of the bulk velocity have the form

$$\mathbf{v}(r, \theta; \mathrm{Re}_\infty) = \mathbf{i} + \ldots, \mathbf{v}(r, \theta; \mathrm{Re}_\infty) = \mathbf{v}_0(r, \theta) + \ldots,$$

where $\mathbf{i}$ is the unit vector in the direction of the incident stream velocity.

Substituting the expansion (2.2) for the function ξ into Eq. (1.2), we have an expansion of the Sherwood number in terms of Re_∞

$$\mathrm{Sh}(\mathrm{Re}_\infty) = \mathrm{Sh}_0 + \mathrm{Re}_\infty \mathrm{Sh}_1 + \ldots, \ \mathrm{Sh}_0 = -\frac{1}{2\pi}\int_S \nabla\xi_0 dS, \ \mathrm{Sh}_1 = -\frac{1}{2\pi}\int_S \nabla\xi_1 dS.$$

We now find the zeroth-order approximation for the inner expansion of the functions ξ, T, corresponding to the case of a plasma at rest. From Eqs. (1.3), (1.4), (2.1), and (2.2) for determining ξ_0 and T_0 we obtain the following equations and boundary conditions:

$$\begin{gathered}\operatorname{div}(\mu_0\nabla T_0) = 0, \ \operatorname{div}(\mu_0\nabla\xi_0) = 0, \ \mu_0 = T_0^n; \\ r = 1, \ \xi_0 = 1, \ T_0 = T_w.\end{gathered} \tag{2.4}$$

In addition, the functions ξ_0 and T_0 must satisfy the conditions for matching with the zeroth-order term of the outer expansion, i.e., should go to 0 and 1 at infinity.

The solution of problem (2.4) is

$$\xi_0 = \frac{1 - T_0}{1 - T_w}, \ T_0 = \left(1 + \frac{T_w^{n+1} - 1}{r}\right)^{1/(n+1)}. \tag{2.5}$$

Now we must find the zeroth-order approximation for the Sherwood number Sh_0, which describes mass transfer in the plasma at rest:

$$\mathrm{Sh}_0 = 2T_w^{-n} f(T_w), f(T_w) = \frac{1}{n+1}\frac{1 - T_w^{n+1}}{1 - T_w}. \tag{2.6}$$

We then go on to construct the next approximation for the Sherwood number Sh. We find the functions $\xi^{(1)}$, $T^{(1)}$. To do this we require the asymptote of the functions ξ_0, T_0 at infinity. From Eq. (2.5) it follows that as $r \to \infty$

$$\begin{gathered}\xi_0 = c_1/r + O(1/r^2), \ c_1 = f(T_w), \ T_0 = 1 + c_2/r + O(1/r^2), \\ c_2 = -(1 - T_w)f(T_w).\end{gathered} \tag{2.7}$$

From Eqs. (1.3), (1.4), (2.1), (2.3), and (2.7) we have the following problem for the functions $\xi^{(1)}$, $T^{(1)}$:

$$\begin{gathered}\mathbf{i}\nabla' T^{(1)} - (1/\sigma)\operatorname{div}'(\nabla' T^{(1)}) = 0, \\ \mathbf{i}\nabla'\xi^{(1)} - (1/\mathrm{Sc})\operatorname{div}'(\nabla'\xi^{(1)}) = 0, \\ r' \to \infty, \ T^{(1)} \to 0, \ \xi^{(1)} \to 0; \ r' \to 0, \ T^{(1)} \sim c_2/r' + \ldots, \\ \xi^{(1)} \sim c_1/r' + \ldots,\end{gathered} \tag{2.8}$$

where ∇', div' are the corresponding differential operators, allowing for radial compression.

The solution of problem (2.8) are the functions

$$\begin{gathered}\xi^{(1)} = (c_1/r')\exp[-(1/2)\,\mathrm{Sc}\, r'(1 - \cos\theta)], \\ T^{(1)} = (c_2/r')\exp[-(1/2)\sigma r'(1 - \cos\theta)].\end{gathered} \tag{2.9}$$

To determine the first term of the inner expansion of the functions ξ, T from Eqs. (1.3), (1.4), (2.1), (2.2), and (2.9) we have the boundary problem

$$\rho_0\mathbf{v}_0\nabla\xi_0 - \frac{1}{\mathrm{Sc}}\operatorname{div}(\mu_0\nabla\xi_1 + \mu_1\nabla\xi_0) = 0, \ \mu_1 = nT_0^{n-1}T_1; \tag{2.10}$$

$$\rho_0\mathbf{v}_0\nabla T_0 - (1/\sigma)\operatorname{div}(\mu_0\nabla T_1 + \mu_1\nabla T_0) = 0; \tag{2.11}$$

$$r=1, \xi_1=0, T_1=0; \qquad (2.12)$$
$$r\to\infty, \xi_1\to -(1/2)c_1\mathrm{Sc}(1-\cos\theta), T_1\to -(1/2)c_2\sigma(1-\cos\theta).$$

From the continuity equation we have the identity

$$\operatorname{div}(\rho_0\mathbf{v}_0)=0. \qquad (2.13)$$

We shall also use the boundary condition for the bulk velocity at the body

$$r=1, \mathbf{v}_0=0. \qquad (2.14)$$

As was mentioned above, the aim of the present analysis is the find expressions for the number Sh_1. For convenience in further calculations we shall introduce the dimensionless Nusselt number Nu, describing the heat flux to the body:

$$\mathrm{Nu}=\frac{1}{2\pi}\int_S \nabla T d\mathbf{S}.$$

Substituting Eq. (2.2) for the function T into the last formula, we obtain an expansion of the Nusselt number in terms of Re_∞

$$\mathrm{Nu}(\mathrm{Re}_\infty)=\mathrm{Nu}_0+\mathrm{Re}_\infty\mathrm{Nu}_1+\ldots, \quad \mathrm{Nu}_0=\frac{1}{2\pi}\int_S \nabla T_0 d\mathbf{S}, \ \mathrm{Nu}_1=\frac{1}{2\pi}\int_S \nabla T_1 d\mathbf{S}.$$

We first find a relation between the numbers Sh_1 and Nu_1. We note that from Eq. (2.4) there follows the identity $\xi_0\operatorname{div}(\mu_0\nabla\xi_1)=\operatorname{div}(\xi_0\mu_0\nabla\xi_1)-\mu_0\nabla\xi_0\nabla\xi_1=\operatorname{div}(\xi_0\mu_0\nabla\xi_1)-\operatorname{div}(\xi_1\mu_0\nabla\xi_0)$.

We multiply Eq. (2.10) by ξ_0, and with the help of the last identity and the identity (2.13) we transform it to the form

$$\frac{1}{2}\operatorname{div}(\xi_0^2\rho_0\mathbf{v}_0)-\frac{1}{\mathrm{Sc}}[\operatorname{div}(\xi_0\mu_0\nabla\xi_1)-\operatorname{div}(\xi_1\mu_0\nabla\xi_0)+\xi_0\operatorname{div}(\mu_1\nabla\xi_0)]=0. \qquad (2.15)$$

We now consider the sphere Σ_R of arbitrary radius $R>1$ and the volume V_R included between this sphere and the surface S of the spherical probe. Integrating Eq. (2.15) over the volume V_R, using the Ostrogradskii–Gauss theorem, and then going to the limit $R\to\infty$, and allowing for the asymptote at infinity, Eqs. (2.7), (2.12) and the boundary conditions on the body, Eqs. (2.4) and (2.12), we obtain

$$I_1-I_2+J=0, \qquad (2.16)$$

where

$$I_1=-\int_S \xi_0\mu_0\nabla\xi_1 d\mathbf{S}=2\pi T_w^n\mathrm{Sh}_1;$$

$$I_2=\lim_{R\to\infty}\int_{\Sigma_R}\xi_1\mu_0\nabla\xi_0 d\boldsymbol{\Sigma}=2\pi\,\mathrm{Sc}\,f^2(T_w); \qquad (2.17)$$
$$J=\lim_{R\to\infty}\int_{V_R}\xi_0\operatorname{div}(\mu_1\nabla\xi_0)\,dV;$$

and the surface element $d\Sigma$ of the sphere Σ_R is directed along the outward normal. The first term in Eq. (2.15) makes no contribution to Eq. (2.16), since for any radius $R>1$ from Eqs. (2.13) and (2.14) we have

$$\int_{\Sigma_R}\rho_0\mathbf{v}_0 d\boldsymbol{\Sigma}=\int_{\Sigma_R}\rho_0\mathbf{v}_0 d\boldsymbol{\Sigma}-\int_S\rho_0\mathbf{v}_0 d\mathbf{S}=\int_{V_R}\operatorname{div}(\rho_0\mathbf{v}_0)\,dV=0 \qquad (2.18)$$

and, therefore,

$$\int_{V_R}\operatorname{div}(\xi_0^2\rho_0\mathbf{v}_0)\,dV=\int_{\Sigma_R}\xi_0^2\rho_0\mathbf{v}_0 d\boldsymbol{\Sigma}-\int_S\xi_0^2\rho_0\mathbf{v}_0 d\mathbf{S}=\xi_0^2(R)\int_{\Sigma_R}\rho_0\mathbf{v}_0 d\boldsymbol{\Sigma}=0. \qquad (2.19)$$

Thus, from Eq. (2.16) we have

$$2\pi T_w^n\mathrm{Sh}_1-2\pi\,\mathrm{Sc}\,f^2(T_w)+J=0. \qquad (2.20)$$

We note that from Eqs. (2.4) and (2.5) there follow the identities $\xi_0 \operatorname{div}(\mu_0 \nabla T_1) = \operatorname{div}(\xi_0 \mu_0 \nabla T_1) - \mu_0 \nabla \xi_0 \nabla T_1 = \operatorname{div}(\xi_0 \mu_0 \nabla T_1) - \operatorname{div}(T_1 \mu_0 \nabla \xi_0)$, $\nabla T_0 = -(1 - T_w) \nabla \xi_0$.

We multiply Eq. (2.11) by ξ_0, and with the help of the last identities and the identity (2.13) we transform it to the form

$$\frac{1}{2}(1 - T_w) \operatorname{div}(\xi_0^2 \rho_0 \mathbf{v}_0) + \frac{1}{\sigma}[\operatorname{div}(\xi_0 \mu_0 \nabla T_1) - \operatorname{div}(T_1 \mu_0 \nabla \xi_0) - (1 - T_w) \xi_0 \operatorname{div}(\mu_1 \nabla \xi_0)] = 0. \tag{2.21}$$

Integrating the last equation over the volume V_R, using the Ostrogradskii–Gauss theorem, and then going to the limit $R \to \infty$, and allowing for the asymptote at infinity (2.7) and (2.12), and the boundary conditions on the body (2.4) and (2.12), we obtain

$$I_3 + I_4 + (1 - T_w) J = 0, \tag{2.22}$$

where

$$I_3 = \int_S \xi_0 \mu_0 \nabla T_1 d\mathbf{S} = 2\pi T_w^n \mathrm{Nu}_1;$$

$$I_4 = \lim_{R\to\infty} \int_{\Sigma_R} T_1 \mu_0 \nabla \xi_0 d\mathbf{\Sigma} = -2\pi\sigma(1 - T_w) f^2(T_w).$$

The integral J in Eq. (2.22) appears in Eq. (2.17). The first term in Eq. (2.21) does not contribute to Eq. (2.22) because of the equality (2.19).

Thus, from Eq. (2.22) we have

$$2\pi T_w^n \mathrm{Nu}_1 - 2\pi\sigma(1 - T_w) f^2(T_w) + (1 - T_w) J = 0.$$

From the last equation and Eq. (2.20) we obtain a relation between the numbers Sh_1 and Nu_1:

$$\mathrm{Sh}_1 = (\mathrm{Sc} - \sigma) T_w^{-n} f^2(T_w) + (1 - T_w)^{-1} \mathrm{Nu}_1. \tag{2.23}$$

It is interesting to note that for a Lewis–Semenov number of $\mathrm{Le} = \sigma/\mathrm{Sc} = 1$ from Eq. (2.23) in the first approximation we obtain the analogy between heat and mass transfer.

We now find the number Nu_1. Using Eqs. (2.4) and (2.10), we transform Eq. (2.11) to the form

$$\rho_0 \mathbf{v}_0 \nabla T_0 - (1/\sigma) \operatorname{div}[\nabla(\mu_0 T_1)] = 0. \tag{2.24}$$

Transforming the last equation with the aid of the identity (2.13), and then integrating it over the volume V_R, using the Ostrogradskii–Gauss theorem and going to the limit as $R \to \infty$, allowing for Eq. (2.18) we obtain

$$J_\infty - J_w = 0, \text{ where } J_\infty = \lim_{R\to\infty} \int_{\Sigma_R} \nabla(\mu_0 T_1) d\mathbf{\Sigma}; \tag{2.25}$$

$$J_w = \int_R \nabla(\mu_0 T_1) d\mathbf{S} = \int_S \mu_0 \nabla T_1 d\mathbf{S} + \int_S T_1 \nabla \mu_0 d\mathbf{S} = 2\pi T_w^n \mathrm{Nu}_1. \tag{2.26}$$

From Eq. (2.4) there follows the relation

$$\operatorname{div}(\nabla T_0^{n+1}) = 0,$$

and therefore we have the identity

$$T_0^{n+1} \operatorname{div}[\nabla(\mu_0 T_1)] = \operatorname{div}[T_0^{n+1} \nabla(\mu_0 T_1)] - \nabla T_0^{n+1} \nabla(\mu_0 T_1) = \operatorname{div}[T_0^{n+1} \nabla(\mu_0 T_1)] - \operatorname{div}[\mu_0 T_1 \nabla T_0^{n+1}].$$

We multiply Eq. (2.24) by T_0^{n+1}, and with the help of the last identity and identity (2.13) we transform it to the form

$$\frac{1}{n+2} \operatorname{div}(T_0^{n+2} \rho_0 \mathbf{v}_0) - \frac{1}{\sigma} \{\operatorname{div}[T_0^{n+1} \nabla(\mu_0 T_1)] - \operatorname{div}[\mu_0 T_1 \nabla T_0^{n+1}]\} = 0.$$

Integrating the last equation over the volume V_R, using the Ostrogradskii–Gauss theorem, and going to the limit as $R \to \infty$, allowing for Eqs. (2.4), (2.7), (2.12), and (2.18) we obtain

$$J_\infty - T_w^{n+1} J_w = (n+1) \lim_{R\to\infty} \int_{\Sigma_R} T_1 \nabla T_0 d\Sigma = 2\pi (n+1)\sigma (1-T_w)^2 f^2 (T_w).$$

Eliminating the quantity J_∞ from this relation with the help of (2.25), we find that $J_W = 2\pi(n+1)^{-1}\sigma(1 - T_W^{n+1})$.

Taking account of Eq. (2.26), we have

$$\mathrm{Nu}_1 = (n+1)^{-1} \sigma T_w^{-n} (1 - T_w^{n+1}). \tag{2.27}$$

Substituting Eq. (2.27) into Eq. (2.23), we obtain

$$\mathrm{Sh}_1 = \mathrm{Sc}\, T_w^{-n} f^2 (T_w) + \sigma T_w^{-n} f(T_w) [1 - f(T_w)]$$

[the function $f(T_W)$ was defined in Eq. (2.6)].

Thus, we obtain the following expansion of the Sherwood number for small values of the Reynolds number:

$$\mathrm{Sh} = 2T_w^{-n} f(T_w) + \mathrm{Re}_\infty T_w^{-n} f(T_w) \{\mathrm{Sc}\, f(T_w) + \sigma [1 - f(T_w)]\} + o(\mathrm{Re}_\infty).$$

Using the solution that has been found, from Eq. (1.2) we find as the first approximation with respect to Re_∞ an expression for the saturation ion current I_i to a spheric probe of radius a:

$$I_i = 8\pi f(T_w) + \mathrm{Re}_\infty 4\pi f(T_w) \{\mathrm{Sc} f(T_w) + \sigma[1 - f(T_w)]\}. \tag{2.28}$$

The dependence of the saturation current I_i on the probe surface temperature T_W for various values of Reynolds number is illustrated graphically in Fig. 1 for the special case when $\mathrm{Sc} = 1$, $\sigma = 0.7$, $n = 0.7$. These graphs show that the saturation ion current depends weakly on the probe surface temperature. In particular, for a plasma at rest ($\mathrm{Re}_\infty = 0$), for a reduction of T_W from 1 to 0.2, the saturation ion current I_i to the spherical probe is reduced by 31%.

3. Influence of Probe Surface Temperature on the Saturation Ion Current. Some topics in the theory of electrical probes in a chemically frozen nonisothermal plasma with variable transport properties were examined in [6-10]. References [6, 7] considered a spherical probe in a plasma at rest. Spherical and cylindrical electric probes in subsonic plasma flow at large Reynolds number were considered in [8]. In [9] formulas were given for the saturation ion current density to wall probes in a similarity boundary layer (the boundary layer on a flat plate, a cone, and in the vicinity of the stagnation point on a blunt body). A spherical electric probe in hypersonic plasma flow in viscous shock layer flow conditions was examined in [10].

On the basis of the results of the above references, and also of those given in Part 2 of this paper, one can conclude that in a specific range of variation of probe surface temperature, the theoretical dependence of the saturation ion current to an electric probe in a chemically frozen plasma on the temperature is weak.

An experimental investigation of the dependence of the volt–ampere characteristics on the probe surface temperature was made in [11-16]. Probes in subsonic flow of low-temperature weakly ionized plasma with $\mathrm{Re}_\infty \sim 1$ were investigated in an experiment [11], and for $\mathrm{Re}_\infty \gtrsim 100$ in [12-16] at atmospheric pressure.

Reference [11] investigated a cylindrical probe of platinum in a plasma of combustion products with sodium additive with temperature $T_\infty^0 \approx 1900$°K, reference [12] investigated a twin probe with electrodes in the form of flat plates of graphite in an argon plasma with potassium additive with temperature $T_\infty^0 = 3000$–4000°K, and reference [13] investigated a twin probe (circular steel electrodes, flush with the surface of a transversely washed circular cylinder in the stagnation line region) in a plasma of combustion products with additive of alkali metals (sodium, potassium, cesium) with temperature $T_\infty^0 \approx 1750$°K. References [14, 15] investigated probes of titanium and zirconium, in the form of a flat plate and a sphere, respectively, in an argon plasma with $T_\infty^0 = 2000$–4000°K, and reference [16] investigated spherical probes of platinum and steel in a plasma of combustion products with additive of alkali metals (sodium, cesium) with temperature $T_\infty^0 \approx 2200$°K.

The result of this work is the conclusion that the ion current (within the limits of accuracy of the experiments) is practically independent of the probe surface temperature, if the temperatures does not exceed a certain value. This value depends on the specific experimental conditions and is ≈1200°K for the operating conditions of [11], ≈1800°K for [12], ≈600°K for [13], ≈1100°K for [14, 15], and ≈800°K for [16].

Thus, the theoretical conclusion derived above is supported by the experimental data.

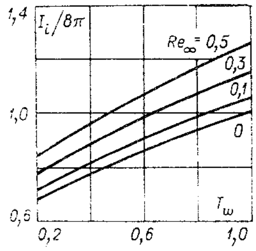

Fig. 1

4. Saturation Current to an Electric Probe of Arbitrary Shape. We now consider subsonic flow of a plasma at low Reynolds number about a probe of arbitrary shape. In view of Part 3 it is sufficient to consider the case of an isothermal plasma in estimating the saturation current. The corresponding boundary-value problem of Eqs. (1.3) and (1.4) with $\rho = 1$, $\mu = 1$ was solved in [2, 3] by the method of matched asymptotic expansions with respect to Re_∞. An expansion of Sh with respect to Re_∞ was obtained in the form

$$\mathrm{Sh} = \mathrm{Sh}_0 + \frac{1}{4}\,\mathrm{Sh}_0^2\mathrm{Sc}\mathrm{Re}_\infty + o\,(\mathrm{Re}_\infty).$$

Therefore, from Eq. (1.2) we obtain in the first approximation with respect to Re_∞ an expression for the saturation ion current I_i to a probe of arbitrary geometric shape:

$$I_i = 4\pi\left(\mathrm{Sh}_0 + \frac{1}{4}\mathrm{Sh}_0^2\mathrm{Sc}\mathrm{Re}_\infty\right). \tag{4.1}$$

It can be seen that the quantity Sh_0 appearing in this relation is connected as follows with the probe capacity C:

$$\mathrm{Sh}_0 = 2C/L$$

(in the Gaussian system of units). It is known that the probe capacity depends only on its geometry; it is either determined theoretically, or it is measured. In particular, for a spherical probe of radius R we have C = R; for a probe in the shape of an ellipsoid of rotation with semiaxes a and b (a is the semiaxis of rotation), for a probe in the form of a slender rod of length L and radius R, and for a disk-shaped probe of radius R one can use the theoretical results of [17]:

$$C = \frac{2\sqrt{a^2-b^2}}{\ln\dfrac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}},\quad a > b;\quad C = \frac{\sqrt{b^2-a^2}}{\operatorname{arctg}\dfrac{\sqrt{b^2-a^2}}{a}},\quad a < b;$$

$$C = \frac{L}{2\ln\dfrac{L}{R}};\quad C = \frac{2R}{\pi}.$$

5. Technique of Probe Diagnostics in a Slowly Moving Plasma. Reference [4] examined experimentally the problem of probe measurements in a slowly moving plasma, when the Reynolds number of the incident flow is on the order of a few units. Because of the lack of an adequate theory for these conditions one uses the "moving probe" technique, in which one can apply static theory [18, 19]. The probe rate of motion is chosen equal to the plasma flow velocity, and then the probe is considered at rest relative to the plasma. The measurements taken in this way agree well with the theory [18, 19].

The following technique was proposed in [4] for matching the probe and plasma flow velocities. With a fixed potential the probe was traversed several times through the plasma in the flow direction. The probe current was measured as a function of its speed of motion. As the probe speed increased from zero to a value somewhat in excess of the flow velocity, the probe current first decreased to some minimum value, and then began to increase. It was postulated in [4] that this minimum corresponds to zero relative velocity of the probe and plasma. The measurements of [4] were taken with a spherical probe.

It follows from Eq. (4.1) that the postulate of [4] is valid not only for a spherical probe, but for a probe of arbitrary shape. Thus, the moving probe technique described above is confirmed theoretically for a probe of arbitrary shape.

We shall show that with the aid of this technique one can determine the concentration of charged particles in the unperturbed flow from the measured saturation ion current without knowing the diffusion coefficient for ions in the neutral gas.

From Eqs. (1.1) and (4.1) we have

$$\left.\frac{d(-I_i^0)}{dv_\infty^0}\right|_{v_\infty^0=0} = \frac{\pi}{2}\,\mathrm{Sh}_0^2 e n_{e\infty}^0 L^2.$$

Therefore, from the slope of the dependence of the saturation current $|I_i^0| = -I_i^0$ on the velocity v_∞^0 of the incident stream at the minimum current point ($v_\infty^0 = 0$) one can determine the concentration of charged particles $n_{e\infty}^0$ without knowing the diffusion coefficient D_{in}^0.

If one needs to calculate the slight influence of the probe surface temperature T_W on the saturation ion current, then, in place of the last formula in the case of a spherical probe one must use a relation deriving from Eqs. (1.1) and (2.28):

$$2\pi f^2(T_w)\, a^2 e n_{e\infty}^0 = \left.\frac{d(-I_i^0)}{dv_\infty^0}\right|_{v_\infty^0=0} - [1 - f(T_w)]\,\sigma\,\frac{(-I_{i0}^0)}{2\,\dfrac{\nu_\infty^0}{a}},$$

where $\nu^0 = \mu^0/\rho^0$ is the kinematic viscosity. From the last relation, according to the slope $\left.\dfrac{d(-I_i^0)}{dv_\infty^0}\right|_{v_\infty^0=0}$ and the value of the dependence ($-I_{i0}^0$) on the velocity $|I_i^0| = -I_i^0$ at the minimum point v_∞^0, one can determine the concentration of charged particles $n_{e\infty}^0$ without knowing the diffusion coefficient D_{in}^0. If the values of D_{in}^0 and v_∞^0 are known, it is convenient to use Eqs. (2.28) and (4.1) to find $n_{e\infty}^0$.

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